Motion Planning Lecture 13

Multi-Robot Motion Planning

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Recap Last Week

Last Week

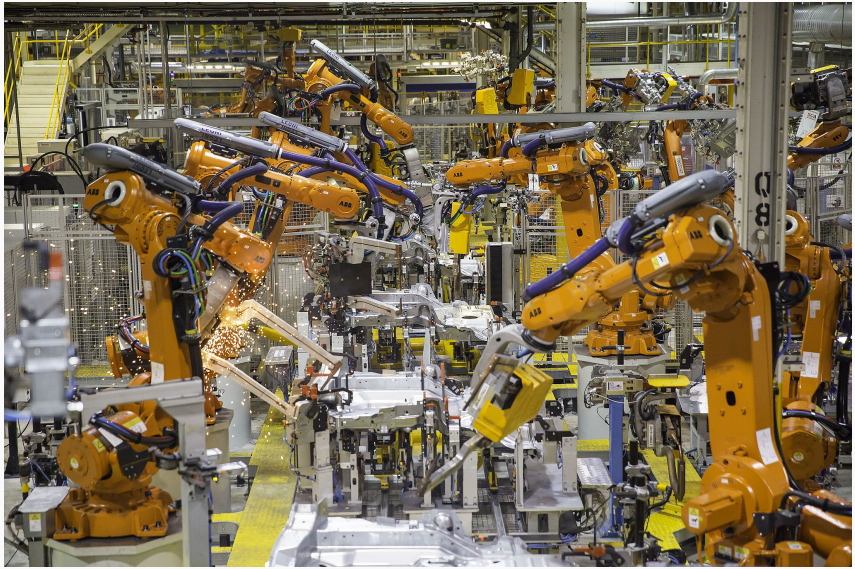
- Optimization-based planning (KOMO)
- Comparison of Search, Optimization, and Sampling
- Hybrid approaches

Today

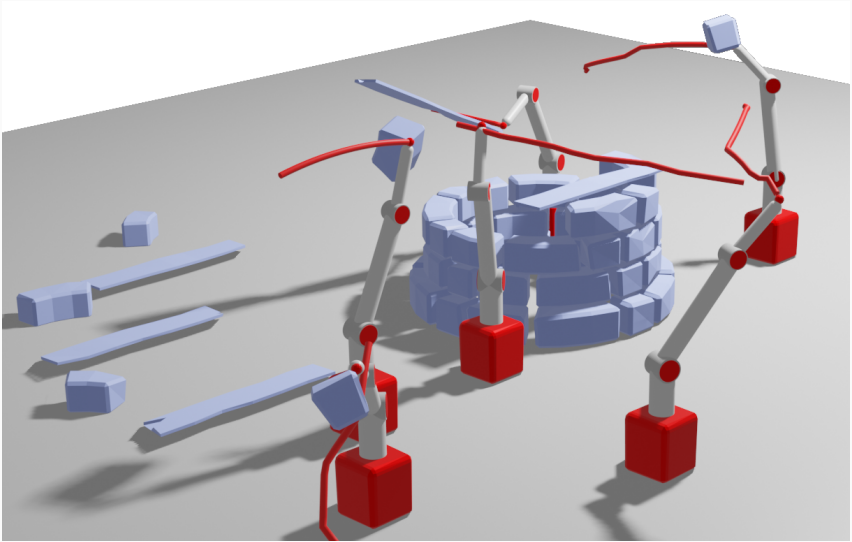
- Multi-robot motion planning
- Planning with manifold constraints

Multi-robot navigation

Multi-robot welding



Multi-robot assembly



Coordination of drones

Multi-robot planning

Multi-robot planning: Coordinate motion of multiple robots acting in the same environment.

Multi-robot planning

Central idea: Multi-robot planning is just planning in the *composite configuration space*.
Robot state spaces X_1, X_2, X_3, Composite space $X = X_1 \times X_2 \times X_3$.

How to adjust primitive methods?

- Sampling
- Interpolation
- Propagation/Steering
- Collision-Checking

Multi-robot planning

Computational complexity is exponential in the number of dimensions.

John Canny, "The complexity of robot motion planning", 1988 [1]

Multi-robot planning

Goal

Find decompositions of the composite configuration space to make multi-robot planning more efficient (while keeping completeness/optimality).

Multi-robot planning: Taxonomy

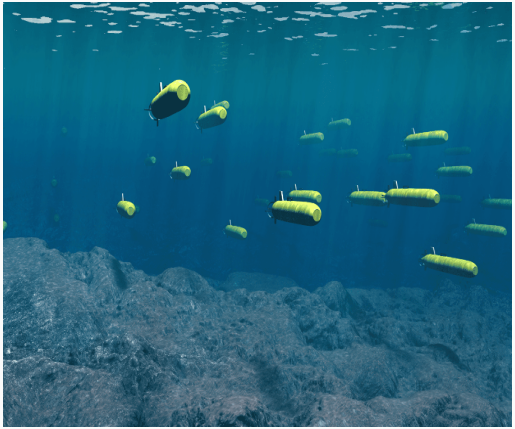
- Are your robots identical? (Homogeneous vs Non-Homogeneous)
- If homogeneous, are the goals interchangeable? (Labeled vs Unlabeled)
- Who controls your robots? (Centralized vs Decentralized)
- What kind of cost do you want to minimize?
  - Makespan (last arrival time)
  - Flowtime (total arrival time)

Multi-robot planning: Taxonomy

- Homogeneous planning
  - Pebbles on a graph
  - Conflict-based search
- Non-Homogeneous planning
  - Prioritized planning (vertical)
  - Decomposed planning (horizontal)
  - M*
  - dRRT/dRRT*

Homogeneous Planning

Homogeneous planning



Homogeneous planning

Also called Multi-agent path finding (MAPF)

- All robots are identical
- All robot state spaces are identical (modulo robot-robot collisions)

Main Idea

Problem can be reduced to pebbles-on-a-graph [2]

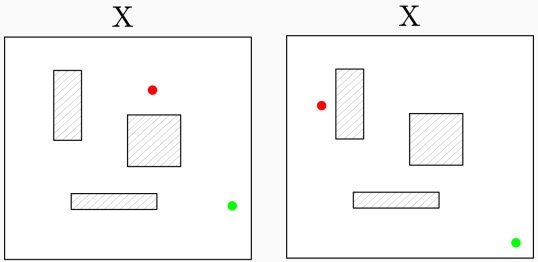
Homogeneous planning: Pebbles on a graph reduction

- Assume we have a homogeneous team of M robots
- Let $X_C = X \times X \times \cdots \times X$ (M times) be the composite state space

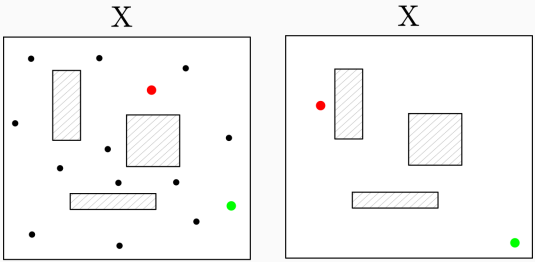
Homogeneous Planning

Example of pebbles on a graph in 2D

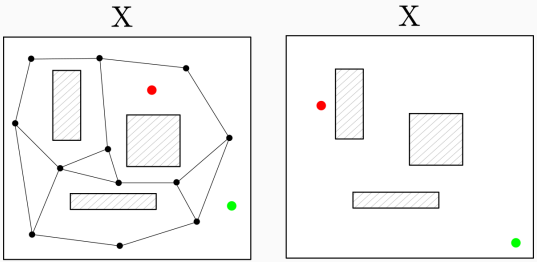
Homogeneous planning: Pebbles on a graph reduction



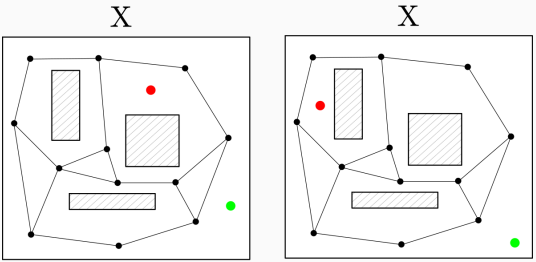
Homogeneous planning: Pebbles on a graph reduction



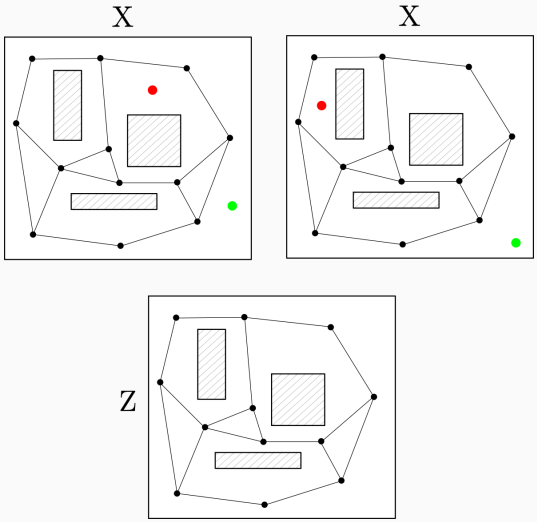
Homogeneous planning: Pebbles on a graph reduction



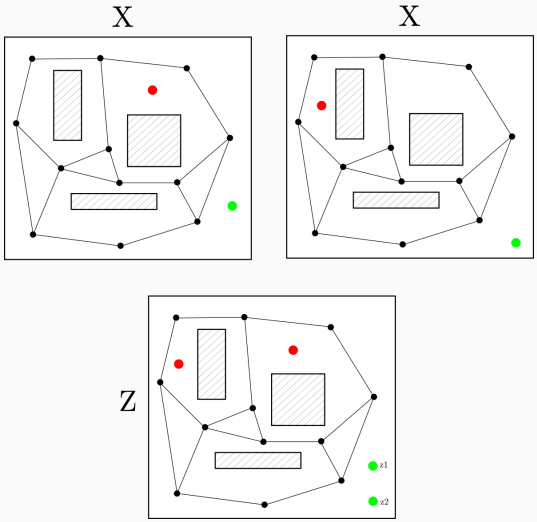
Homogeneous planning: Pebbles on a graph reduction



Homogeneous planning: Pebbles on a graph reduction



Homogeneous planning: Pebbles on a graph reduction



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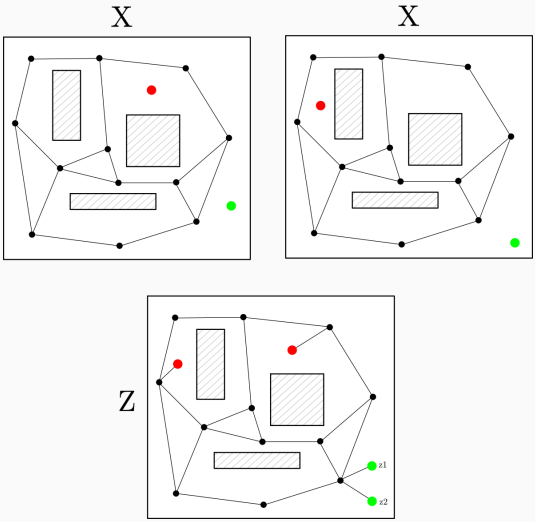
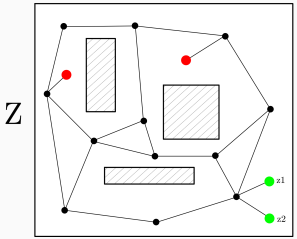


Homogeneous planning: Pebbles on a graph reduction



Pebbles-on-a-graph Problem

Move M pebbles on a graph from a start arrangement to a goal arrangement. Pebbles are not allowed to collide.

see e.g. Kornhauser, "Coordinating Pebble Motion on Graphs" (1984) [3]

Homogeneous Planning

Theory of pebbles on a graph

Homogeneous planning: Pebbles on a graph reduction

Let $X_C = X \times \ldots \times X$ be a homogeneous planning problem for M robots with M start/goal pairs $(x_I^1, x_G^1), \ldots, (x_I^M, x_G^M)$.

Pebbles-on-a-graph reduction

- Create a *single* roadmap G on X.
- Add start/goal pair vertices to X and connect to G
- Solve this simultaneously while taking robot-robot collisions into account.

Homogeneous planning: Pebbles on a graph reduction

Pebbles-on-a-graph

Let $G = (V, E)$ be a graph on X. Let us assume that there are M pebbles (or agents), which start at vertices $s_1, \ldots, s_M$ (start arrangement) and need to go to vertices $g_1, \ldots, g_M$ (goal arrangement). [3]

Approaches to pebbles-on-a-graph

- Push and Swap [4]
- Integer Linear Programming [5]
- Conflict-based search [6]

Homogeneous planning: Pebbles on a graph reduction

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Homogeneous Planning

Conflict-based search

Homogeneous planning: Solving pebbles on a graph

Conflict-based search I

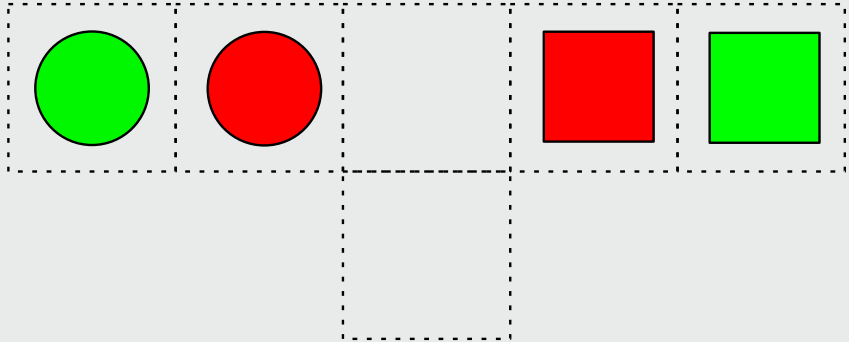
- Assume each pebble has two moves
- Wait at current vertex
- Move to adjacent vertex
- Goal: Find set of moves to reach goal arrangement.

Conflict-based search II

- Key idea: Resolve collisions one-by-one
- First, plan for each pebble individually
- Second, resolve collisions
  - Assume pebbles A,B collide at time T at vertex V
  - Add collision constraints to problem
  - Either (1) A should not be V at time T
  - Or (2) B should not be V at time T
  - This creates a *constraint tree*
  - Pick next entry based on "Best cost" and "First collision first" tie-breaker.

Homogeneous planning: Solving pebbles on a graph

Constraint Tree Example



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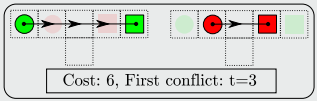

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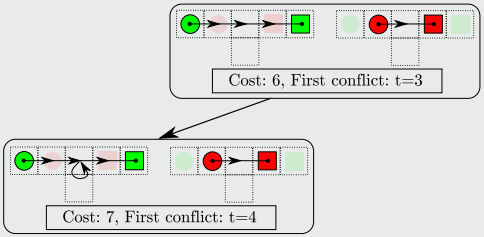

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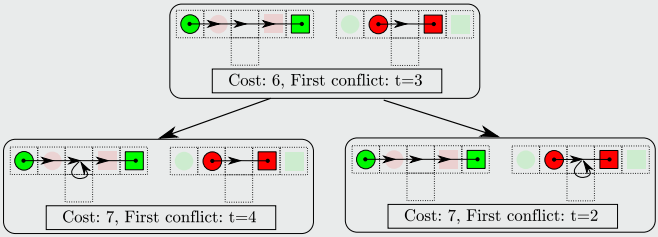

Homogeneous planning: Solving pebbles on a graph

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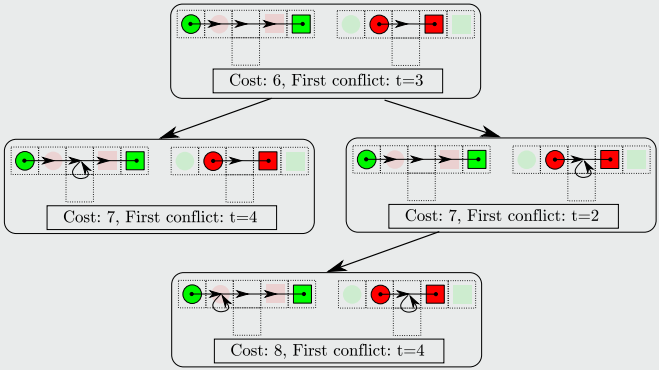

Homogeneous planning: Solving pebbles on a graph

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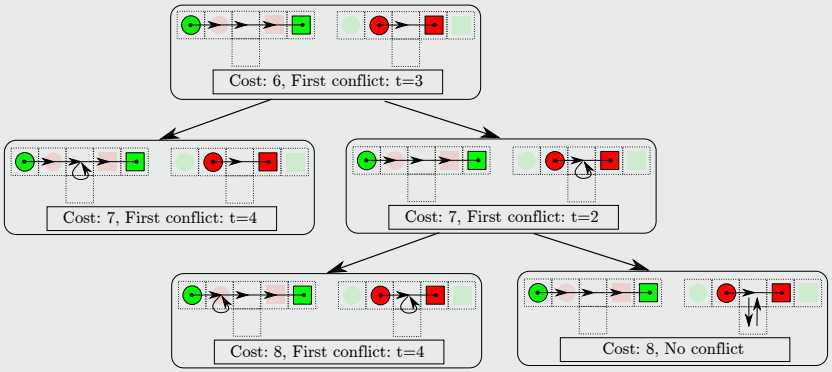

Optimality

Theorem

Conflict-based search returns the optimal solution.

Proof

Part 1: Proof that no valid paths are removed by conflict resolution.

Part 2: Proof that a goal node path has the lowest cost.

Optimality

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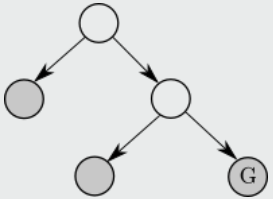


Best-first search with open nodes (grey), closed nodes (white), and goal nodes (G).

Optimality

Theorem

Conflict-based search returns the optimal solution.

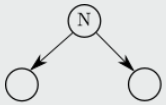
Proof I: No valid paths are removed by conflict resolution.

Let N be a node in the constraint tree, and $CV(N)$ be all valid and consistent paths at N.

(1) If we add a constraint at N, we split into nodes N_1, N_2.

(2) A valid path needs to be either in N_1 or in N_2 or it is invalid.

(3) Therefore, $CV(N)$ is split into $CV(N_1)$ and $CV(N_2)$ (a valid path is either in N_1 or in N_2).



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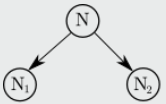


Optimality

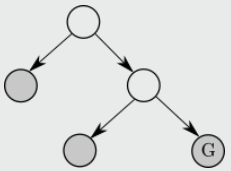
Theorem

Conflict-based search returns the optimal solution.

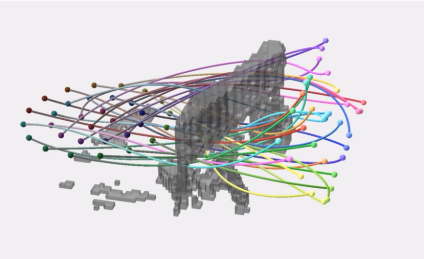
Proof II: A goal node path has the lowest cost.

(1) Let us assume that we reached a goal node G with cost $c(G)$.
(2) Let p be an arbitrary valid path. Then p must be in an open node N. Then it is lower bounded by the best cost at N, i.e. $c(N(p)) \leq c(p)$.
(3) Since we used best-first search, the cost at G is the lowest of all open nodes. Therefore

$$c(G) \leq c(N(p)) \leq c(p)$$

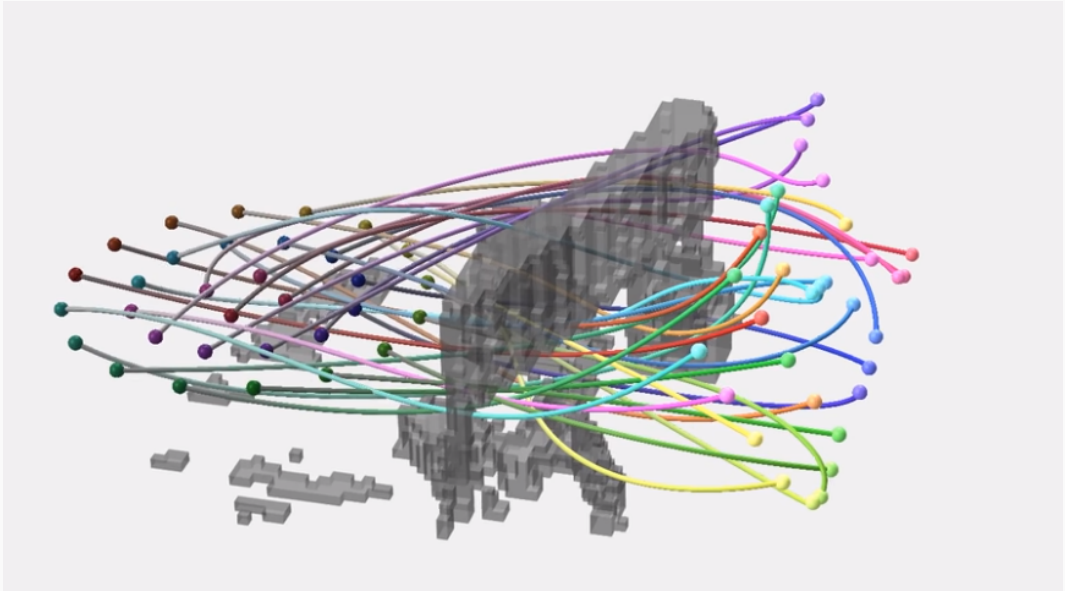


Homogeneous multi-robot planning



Example of 32 drones on composite state space $\mathbb{R}^{192}$.
Using homogeneous multi-robot planning: Computation time is 50s (roadmap generation on $\mathbb{R}^6$) plus 0.8s (conflict annotation) plus 0.5s (conflict-based search) plus 6.5s (optimization) [7]

Homogeneous multi-robot planning



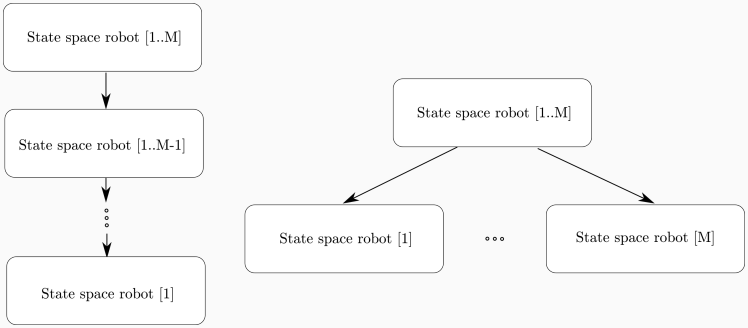
Non-Homogeneous Multi-robot Motion Planning

Non-homogeneous planning

Non-Homogeneous Multi-Robot Planning

- All state spaces differ
- No easy reduction possible
- Can usually not efficiently be solved in composite state space

Non-Homogeneous Multi-Robot Planning

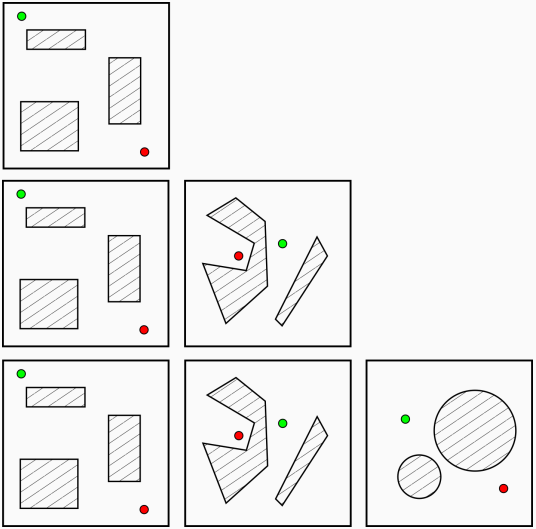


Main approaches

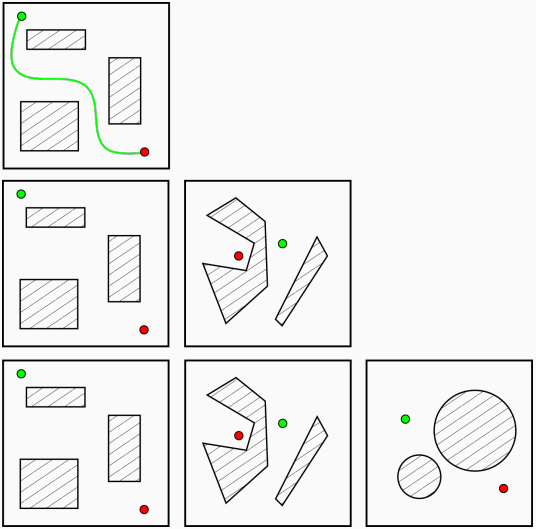
- Prioritized (also greedy, or "vertical")
- Decomposition (also "horizontal")

Prioritized Multi-Robot Motion Planning

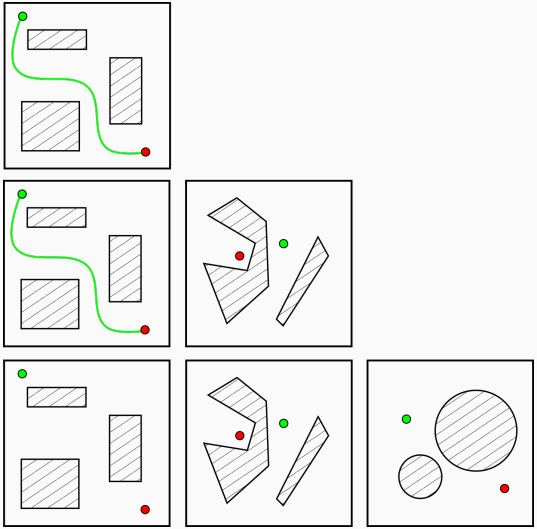
Non-Homogeneous Multi-Robot Planning: Prioritized



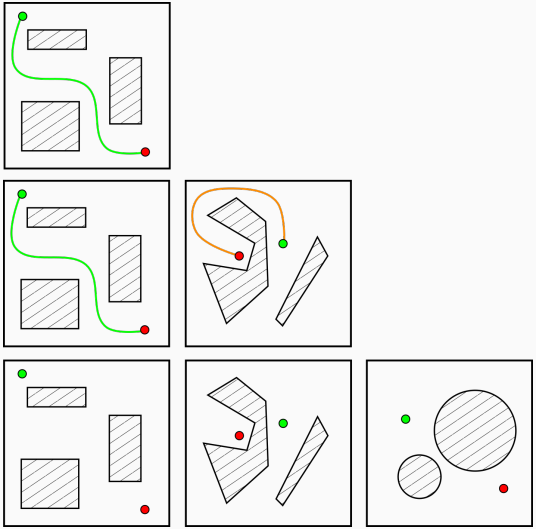
Non-Homogeneous Multi-Robot Planning: Prioritized



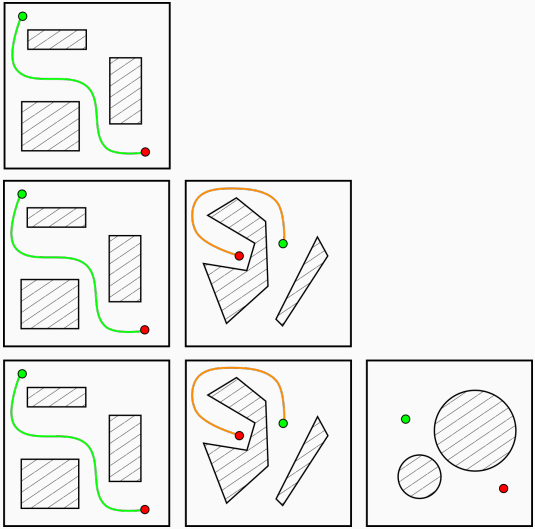
Non-Homogeneous Multi-Robot Planning: Prioritized



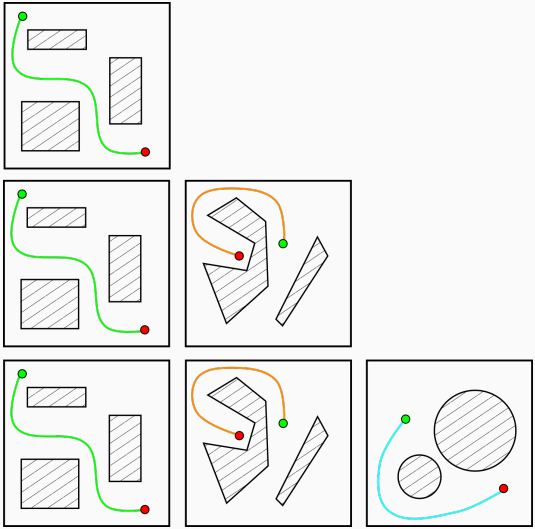
Non-Homogeneous Multi-Robot Planning: Prioritized



Non-Homogeneous Multi-Robot Planning: Prioritized



Non-Homogeneous Multi-Robot Planning: Prioritized

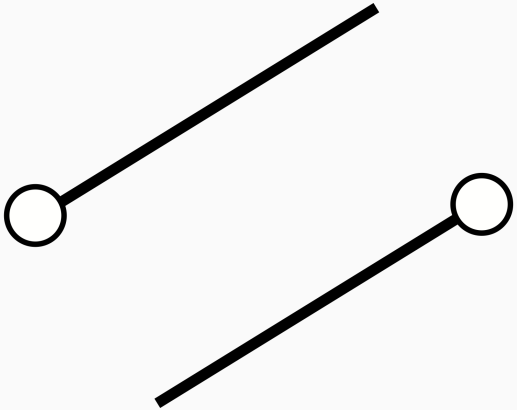


Non-Homogeneous Multi-Robot Planning

- Let $X = Y \times Z$ be the composite state space.
- Prioritized planning: Find path on Y, then use this as a constraint on X
- Constraint can be modelled as a *path restrictions*.

Non-Homogeneous Multi-Robot Planning

Consider two 1-d robots with state space S^1 (the circle).



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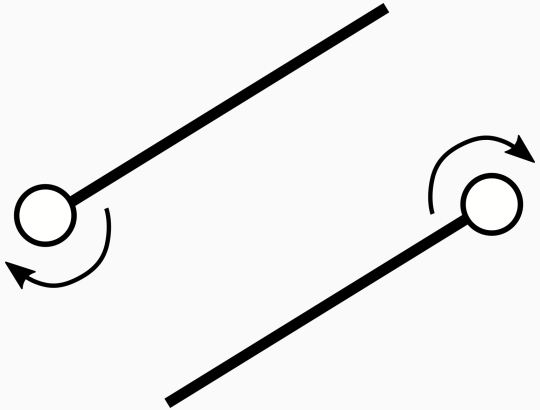
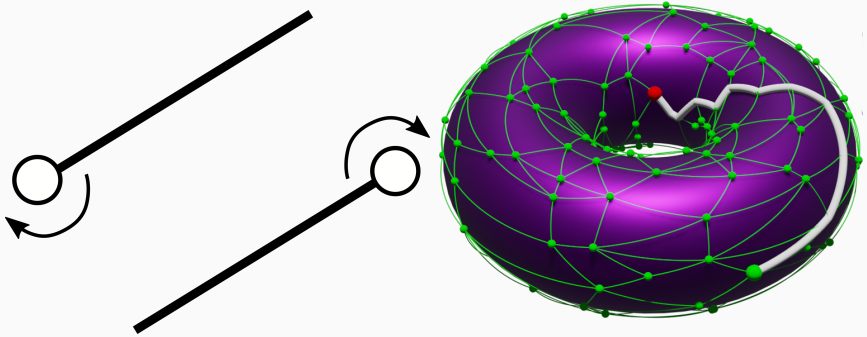
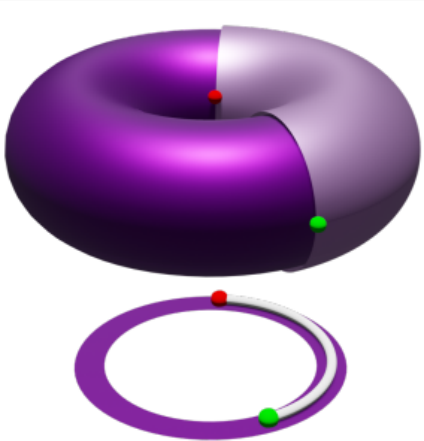


Composite state space is the torus T^2.



Prioritized planning can be seen as projection T^2 down to S^1.
A path on S^1 induces a path restriction on T^2.



Composite State Space

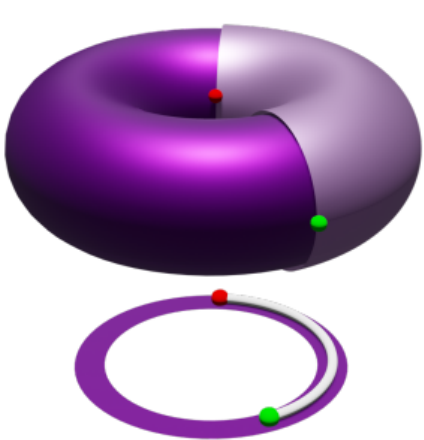
Let $X = Y \times Z$ be the (composite) state space.

Projection

A projection $\pi : X \to Y$ is a mapping from X to Y. Example: $\pi_{T^2} : (\theta_1, \theta_2) \to (\theta_1)$

Base space

Given a projection $\pi : X \to Y$, we call Y the *base space*.



Restriction

Let $X = Y \times Z$ and $\pi : X \to Y$ be a projection.

Given a subset U of Y, we call

$$\pi^{-1}(U) = \{x \in X \mid \pi(x) \in U\} \qquad (1)$$

a restriction.

Path Restriction

Let $p : [0,1] \to Y$ be a path with image $U = p([0,1]) \subset Y$.

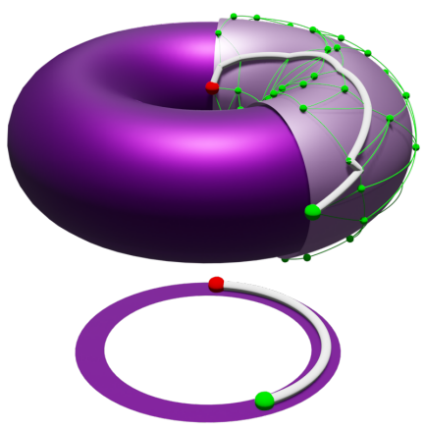
Then $\pi^{-1}(U)$ is called the *path restriction*.

Restriction sampling

Let $\pi_{T^2} : (\theta_1, \theta_2) \to (\theta_1)$ be the projection $\pi : T^2 \to S^1$, and p be a path on S^1.

Restriction sampling:

1. Sample an element y in $p([0, 1])$.
2. Sample an element of $\pi^{-1}(y)$

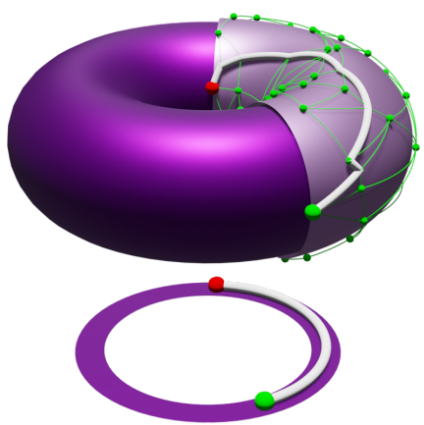


Non-Homogeneous Multi-Robot Planning

Guarantees

Probabilistic completeness?

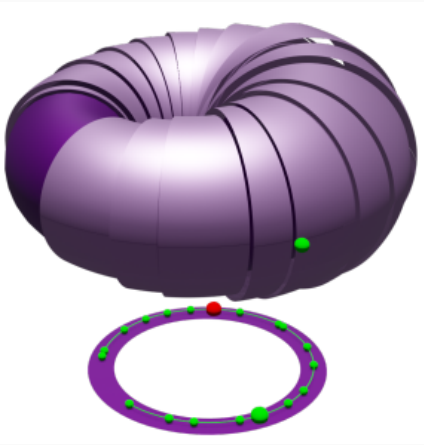
Asymptotic optimality?



Non-Homogeneous Multi-Robot Planning

Probabilistic completeness and asymptotic optimality is possible. Requires two changes.

- Replace path restriction sampling with *graph restriction sampling*
- Continue sampling on base space(s).



A Orthey, S Akbar, M Toussaint, "Multilevel Motion Planning: A Fiber Bundle Formulation", IJRR, 2023 [8]

Non-Homogeneous Multi-Robot Planning

GreedyPrioritizedPlanner(x_I, X_G, $X_1, \ldots, X_K$)

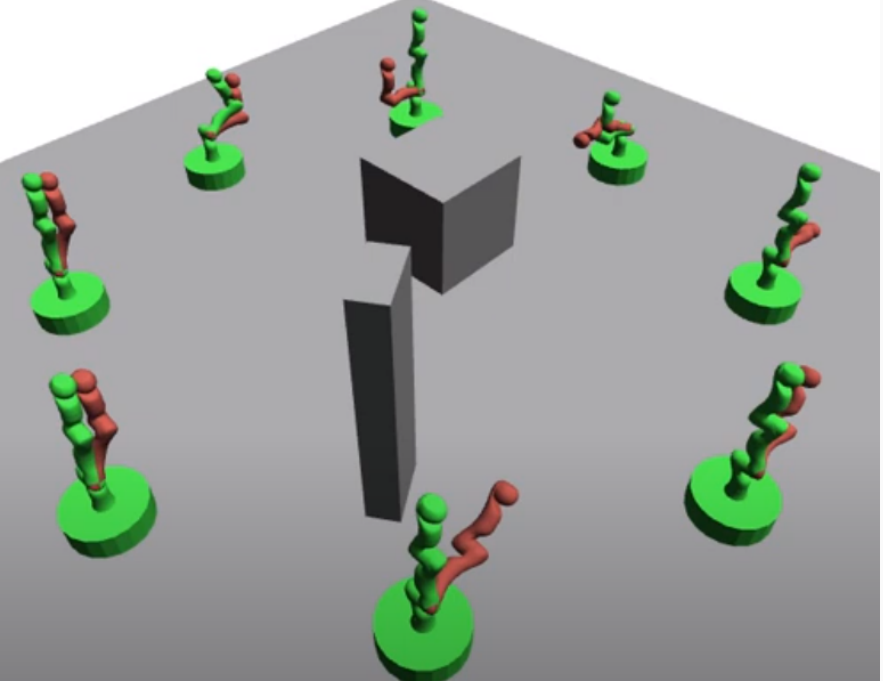
1. For $k = 1$ to K do
2. While not terminated (X_k)
3. Grow(X_k)
4. p_k = GetPath(X_k)
5. SetPathRestriction(p_k, X_{k+1})

Non-Homogeneous Multi-Robot Planning

PrioritizedPlanner(x_I, X_G, $X_1, \ldots, X_K$)

1. $Q \leftarrow \emptyset$ (priority queue)
2. For $k = 1$ to K do
3. $\quad Q$.push(X_k)
4. $\quad$ While not terminated (X_k)
5. $\quad\quad X_{select} \leftarrow Q.pop()$
6. $\quad\quad$ Grow(X_{select})
7. $\quad\quad Q$.push(X_{select})

A Orthey, M Toussaint, "Rapidly-Exploring Quotient-Space Trees: Motion Planning using Sequential Simplifications", ISRR, 2019 [**Orthey2019ISRR**]



Advantages of Prioritized Planning

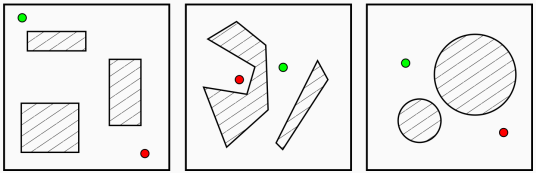
- Paths/graphs on base space as necessary condition on solution (same principle as admissible heuristics/A*)
- Fast if robots are near-decomposable (robot-robot collisions are rare)

Disadvantages of Prioritized Planning

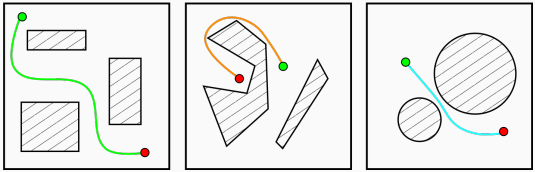
- Ordering of robots needs to be provided
- Unclear which spaces to grow first

Decomposed Multi-Robot Motion Planning

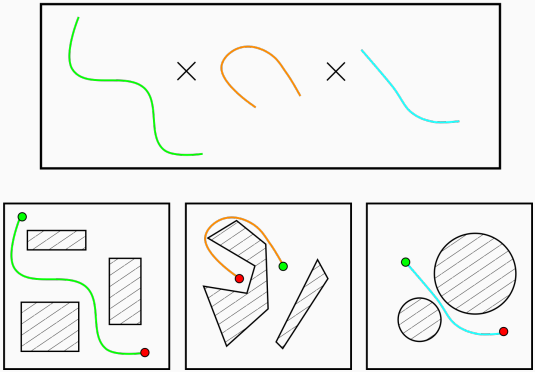
Non-Homogeneous Multi-Robot Planning: Decomposed



Non-Homogeneous Multi-Robot Planning: Decomposed



Non-Homogeneous Multi-Robot Planning: Decomposed



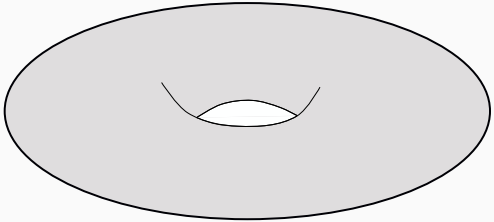
Non-Homogeneous Multi-Robot Planning: Decomposed

Intersection of path restrictions

- Let $X = Y \times Z$ be the composite state space.
- Decomposed planning: Use projections $\pi_Y : X \to Y$ and $\pi_Z : X \to Z$.
- Find path p_1 on Y, and find path p_2 on Z.
- Compute path restrictions $R_1 = \pi_Y^{-1}(p_1)$ and $R_1 = \pi_Z^{-1}(p_2)$.
- Define motion planning problem in intersection $X_R = R_1 \cap R_2$.

Non-Homogeneous Multi-Robot Planning: Decomposed

Example: Path restrictions on $T^2 = S_1^1 \times S_2^1$ with $\pi_1 : T^2 \to S_1^1$, and $\pi_2 : T^2 \to S_2^1$.



Non-Homogeneous Multi-Robot Planning: Decomposed

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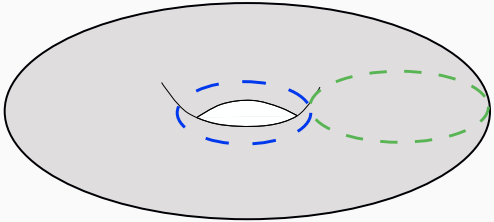

Non-Homogeneous Multi-Robot Planning: Decomposed

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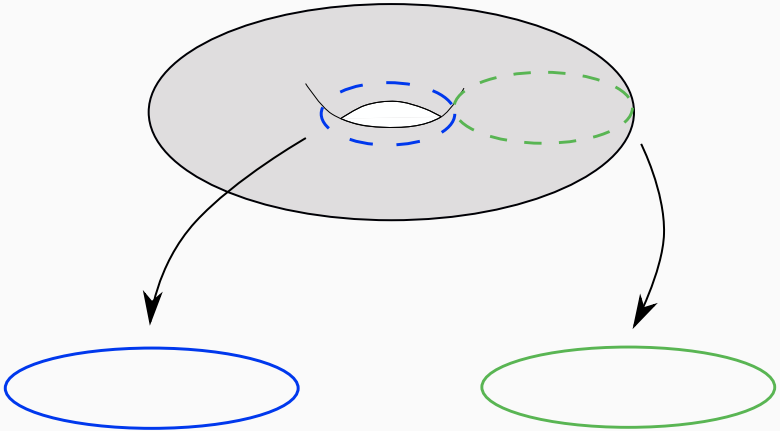

Non-Homogeneous Multi-Robot Planning: Decomposed

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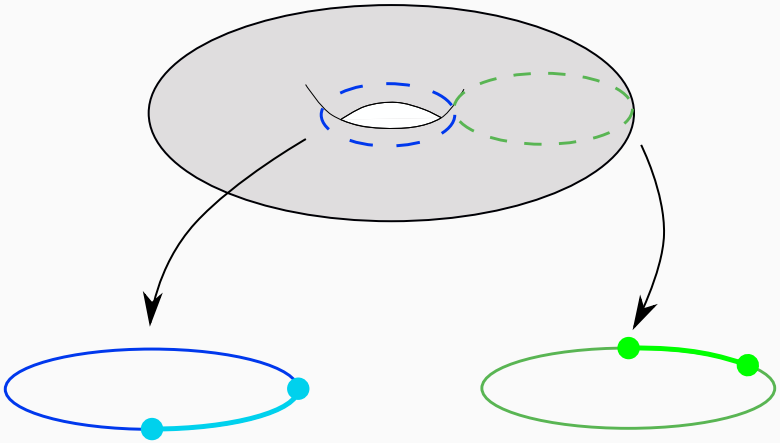

Non-Homogeneous Multi-Robot Planning: Decomposed

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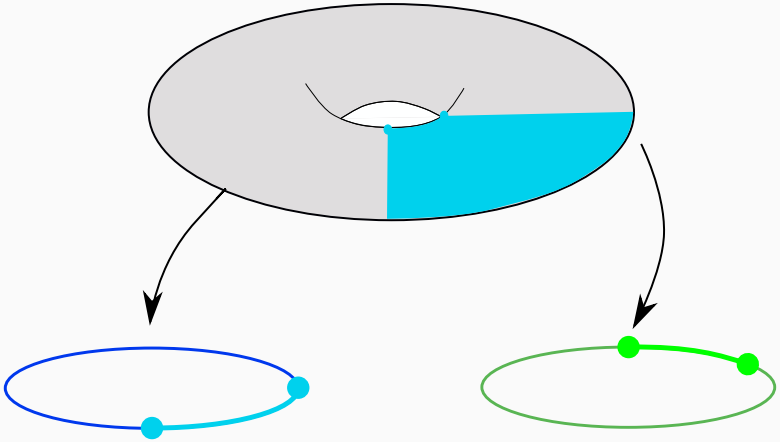

Non-Homogeneous Multi-Robot Planning: Decomposed

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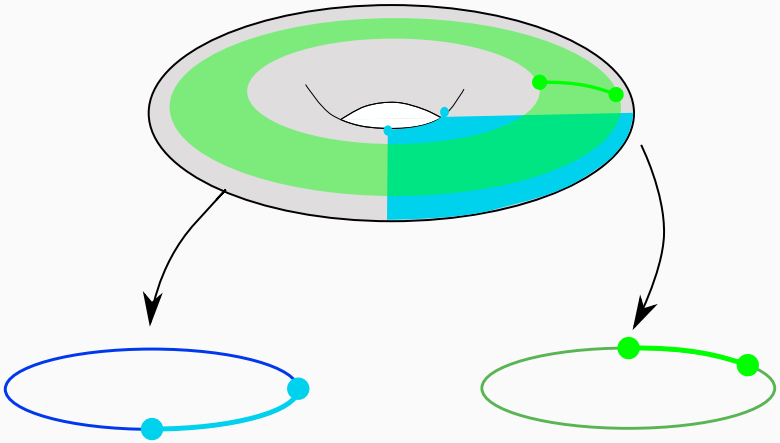

Non-Homogeneous Multi-Robot Planning: Decomposed

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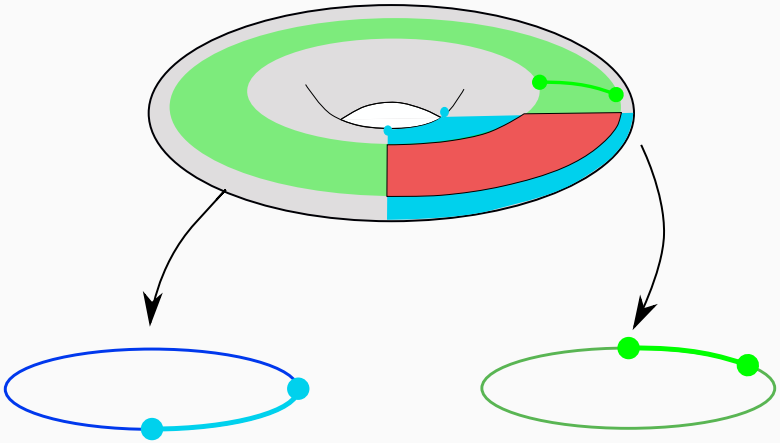

Non-Homogeneous Multi-Robot Planning: Decomposed

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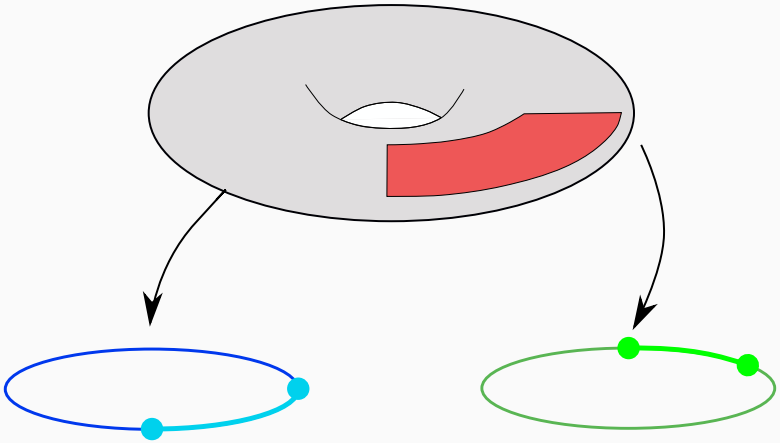

Non-Homogeneous Multi-Robot Planning: Decomposed

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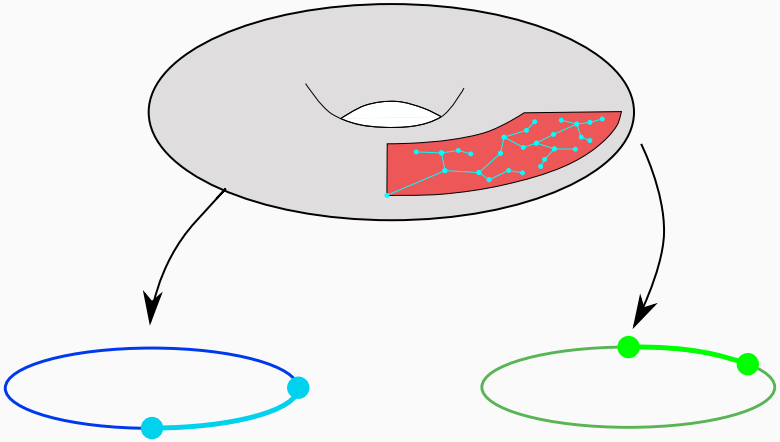


Non-Homogeneous Multi-Robot Planning: Decomposed

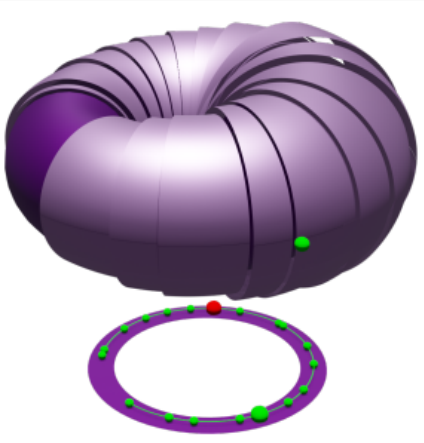
Intersection of path restrictions

- Intersection of path restrictions is equivalent to space of path reparameterizations ($s : [0, T] \rightarrow [0, 1]$).
- Finding a path over this intersection is called *path coordination* [9]

Is this optimal?

Non-Homogeneous Multi-Robot Planning: Decomposed

Adding completeness:
Replace path restriction with graph restriction.
$\Rightarrow$ *Graph coordination*



Non-Homogeneous Multi-Robot Planning: Decomposed

Decomposed planning

- Step 1: Compute individual graphs on component state spaces
- Step 2: Consider the (implicit) product of graphs on the composite state space
- Step 3: Expand edges by optimistically follow shortest paths on the component state spaces
- Step 4: If conflicts arise, backtrack around conflict areas
  - M*: Using cost-to-go estimate [10]
  - dRRT/dRRT*: Using directional oracle [11]

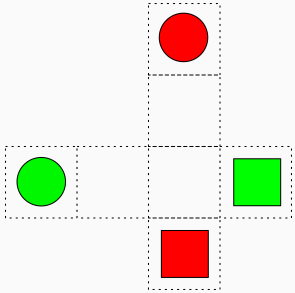
M*

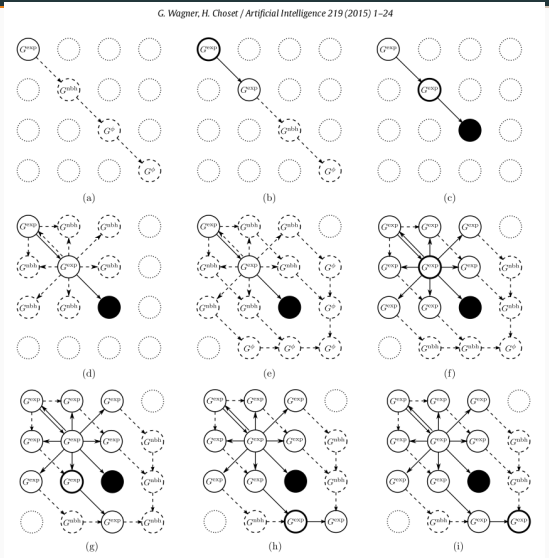
Non-Homogeneous Multi-Robot Planning: Decomposed

Conflict-resolution in graph coordination

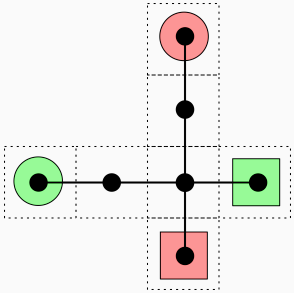
M* works like A*, using as admissible heuristic the cost-to-go of individual spaces

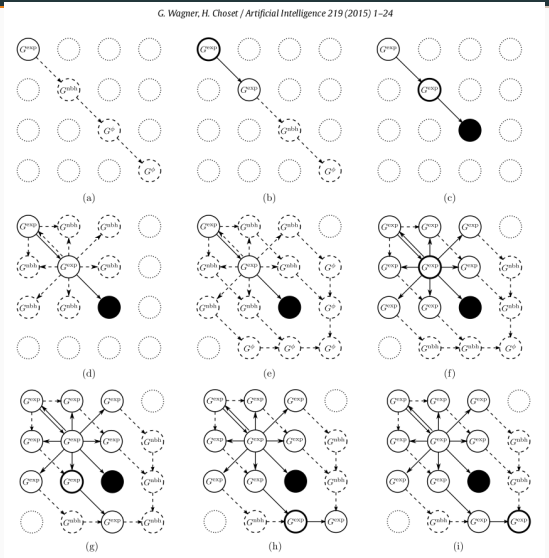
Non-Homogeneous Multi-Robot Planning: Decomposed





Non-Homogeneous Multi-Robot Planning: Decomposed





Non-Homogeneous Multi-Robot Planning: Decomposed

M*

M* is efficient, because it optimistically exploits implicit graph.

- Individual shortest paths are admissible heuristics from individual robots.
- M* combines those admissible heuristics in an optimal way

Drawback: Inefficient for higher dimensions (number of neighbors grows exponentially)

Discrete RRT (dRRT)

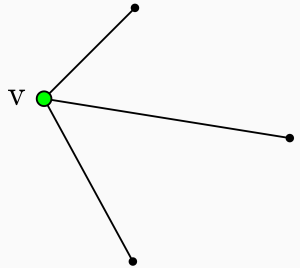
Non-Homogeneous Multi-Robot Planning: Decomposed

dRRT

Discrete-RRT (dRRT) works like M*, but uses an oracle function to pick best neighbor

- Oracle works for euclidean spaces, i.e. planning in $\mathbb{R}^n$.
- Oracle gives you a faster (approximate) ordering of neighbors

Non-Homogeneous Multi-Robot Planning: Decomposed



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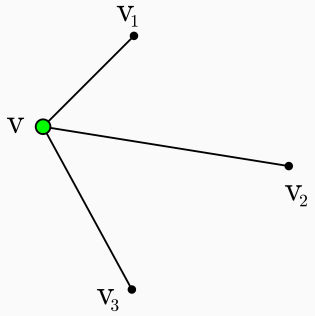
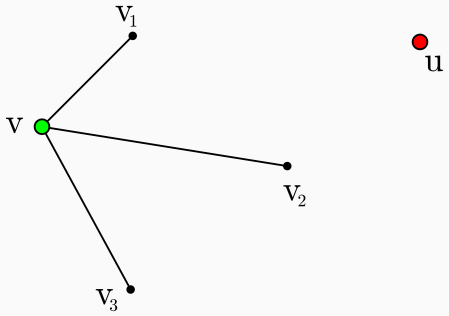


Non-Homogeneous Multi-Robot Planning: Decomposed



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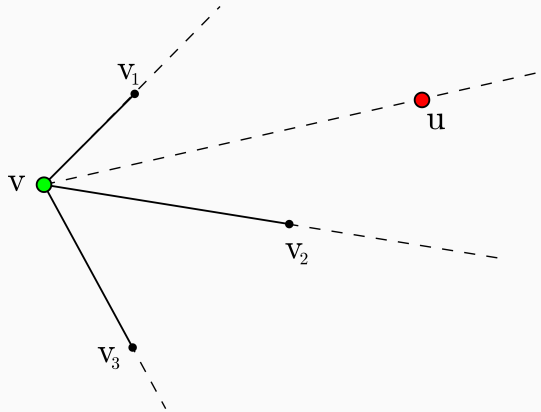
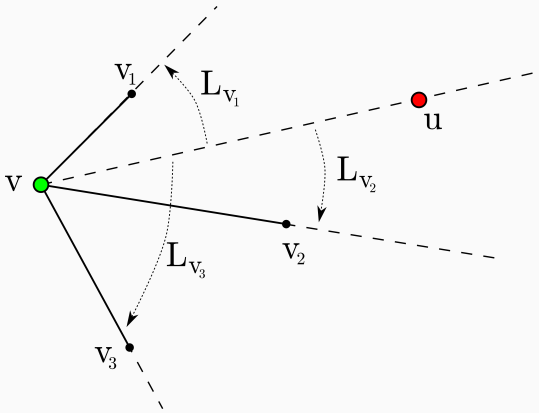


Non-Homogeneous Multi-Robot Planning: Decomposed



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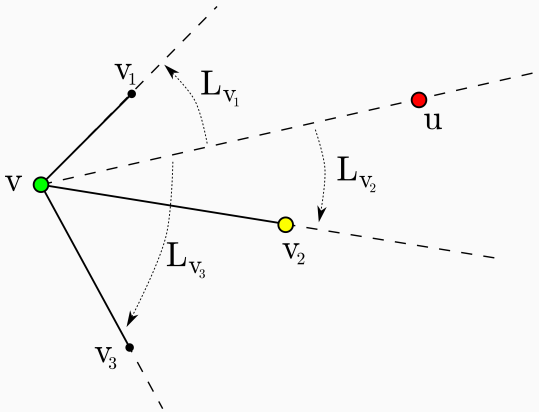


Non-Homogeneous Multi-Robot Planning: Decomposed

Directional oracle in dRRT

Given vertex v, neighbor edges $(v, v') \in E$, and random point u in $\mathbb{R}^n$

$$O(v, u) = \underset{v'}{\operatorname{argmin}}\{\theta_v(u, v') \mid (v, v') \in E\}$$

Summary multi-robot approaches

Multi-Robot Planning: Homogeneous planning

Homogeneous planning: Advantages

- Superior reduction if all robots are equivalent
- Conflict-based search (optimal)
- Scales well with number of robots

Homogeneous planning: Disadvantages

- Cannot be applied to non-homogeneous teams

Multi-Robot Planning: Prioritized non-homogeneous planning

Prioritized multi-robot planning

- Replaces original problems with a sequence of simpler problems
- Solutions to simpler problems provide admissible heuristics

Prioritized multi-robot planning: Disadvantages

- Requires ordering

Multi-Robot Planning: Decomposed non-homogeneous planning

Decomposed multi-robot planning

- Planning for each robot individually (could be done in parallel)
- Combine solutions into one tensor graph on composite state space
- If environment is static, graphs could be precomputed

Decomposed multi-robot planning: Disadvantages

- Individual graphs need to be dense to give good solutions

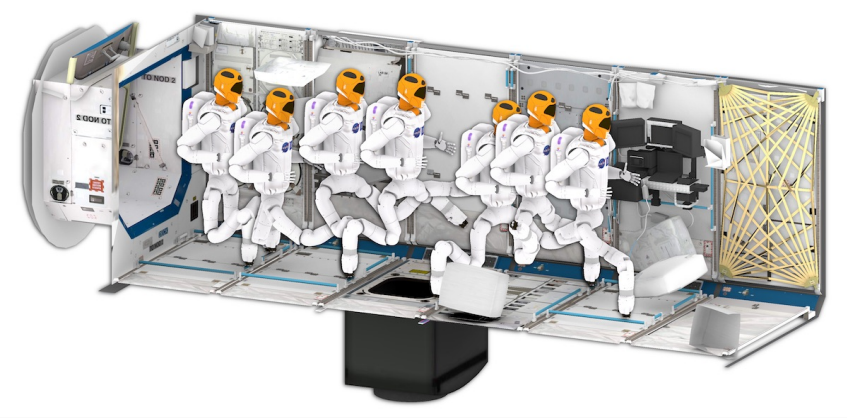
Planning with Manifold Constraints

Planning with Manifold Constraints

Manifold constraints

Constraints on a state space which remove effective degrees of freedom

Planning with Manifold Constraints: Contacts



Planning with Manifold Constraints: Grasping

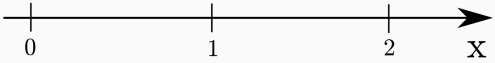
Planning with Manifold Constraints: Surface welding

Zero-measure sets

Lebesgue-measure

- Measure as generalization of volume
- Lebesgue-measure (or box measure): n-dimensional volume, corresponds to length (1D), area (2D), volume (3D).

Planning with Manifold Constraints



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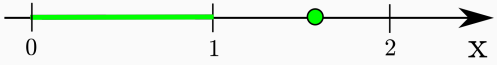

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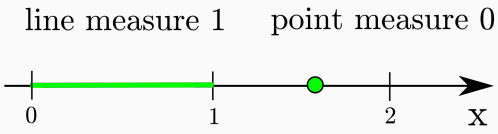

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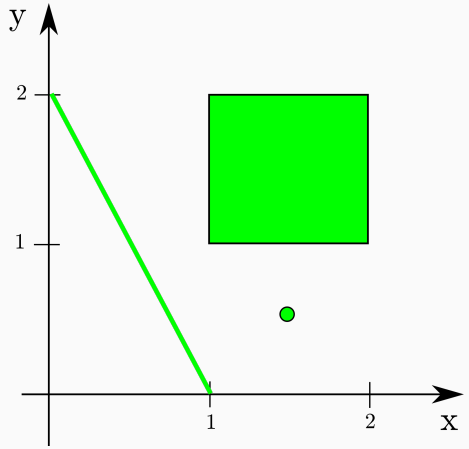

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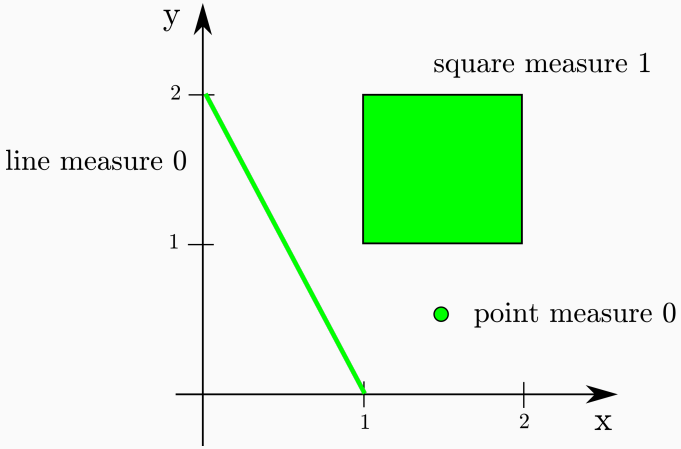

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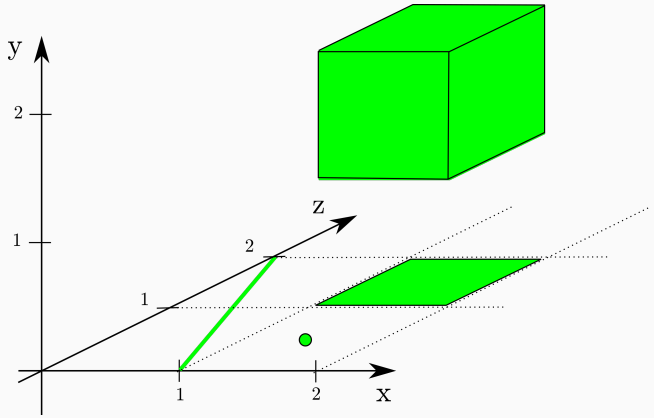

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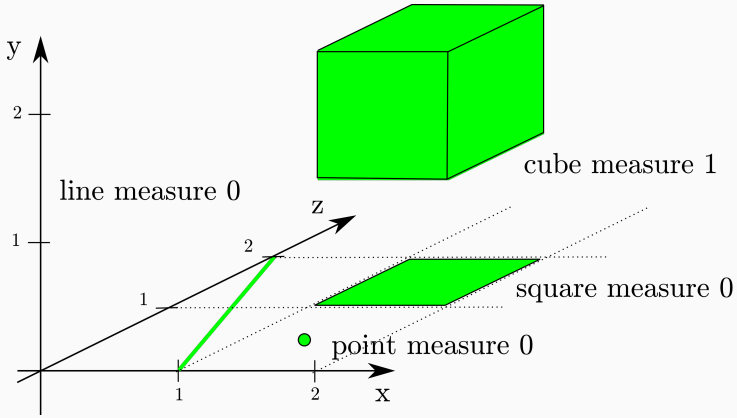
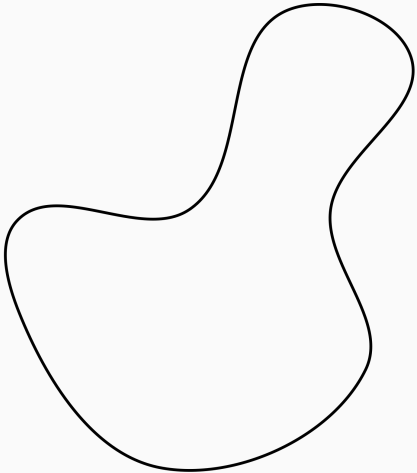


Construction Lebesgue-measure for n-dimensional box

- Define an n-dimensional box $B = \prod_{i=1}^{n}[a_i, b_i]$ (this is the Cartesian product of intervals $[a_i, b_i]$, such that $a_i < b_i$)
- Define Lebesgue-measure as

$$\mu(B) = vol(B) = \prod_{i=1}^{n}(b_i - a_i). \tag{2}$$

Planning with Manifold Constraints



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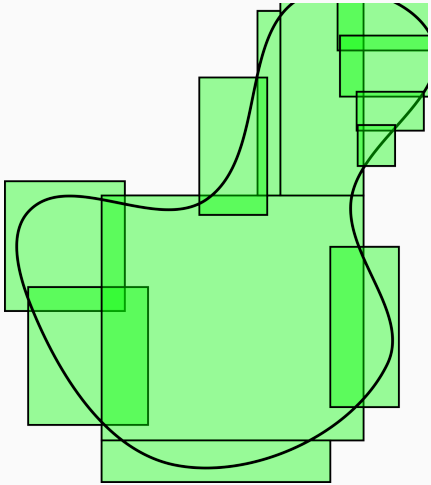


Construction Lebesgue-measure

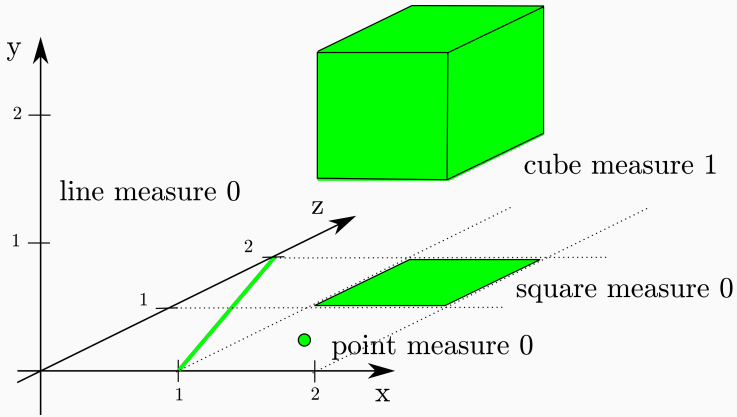
- Let $U \subset \mathbb{R}^n$ be any set.
- Define Lebesgue-measure as the minimal volume over all sets of boxes which cover U.

$$\mu(U) = vol(U) = \inf_{C} \left\{ \sum_{B \in \mathbf{C}} vol(B) \right\},$$

with $\mathbf{C}$ being a collection of boxes covering U.

Zero-measure sets

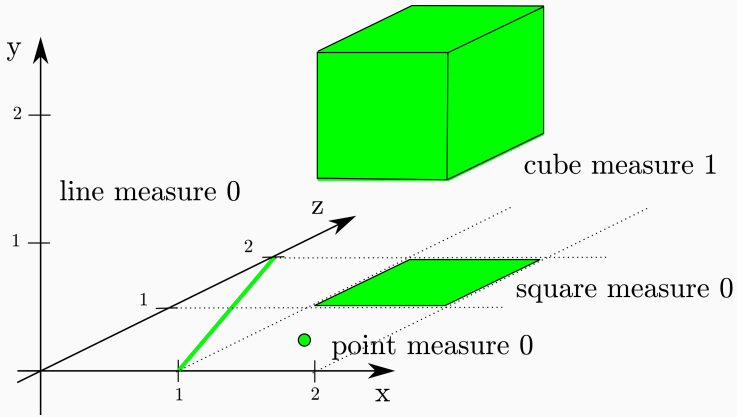
- Lebesgue measure μ is defined relative to dimensionality of space.
- Let n be the dimensionality of the state space.
- A set has zero measure if $d \leq n - 1$, whereby d is the dimension of the set (minimal number of parameters to describe it).



Zero-measure sets

- Probability of sampling in a zero-measure set is zero.

Planning with Manifold Constraints



Positive-measure sets

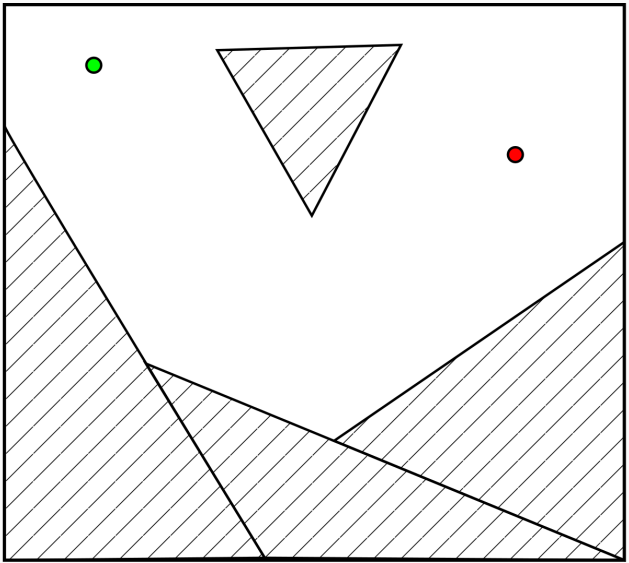
- If $U \subseteq X$ is a set with measure $\mu(U) > 0$, then the probability of sampling U with uniform sampling of X is one.

References

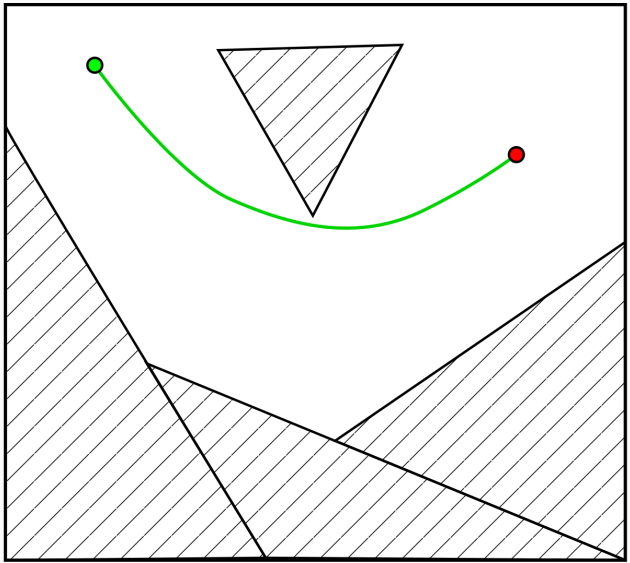
- LaValle, Planning Algorithms, 2006 (Sec. 5.1.3)
  `http://lavalle.pl/planning/node190.html`
- Measure theory `https://en.wikipedia.org/wiki/Measure_(mathematics)`

Planning with zero-measure sets

Planning with Manifold Constraints



Planning with Manifold Constraints



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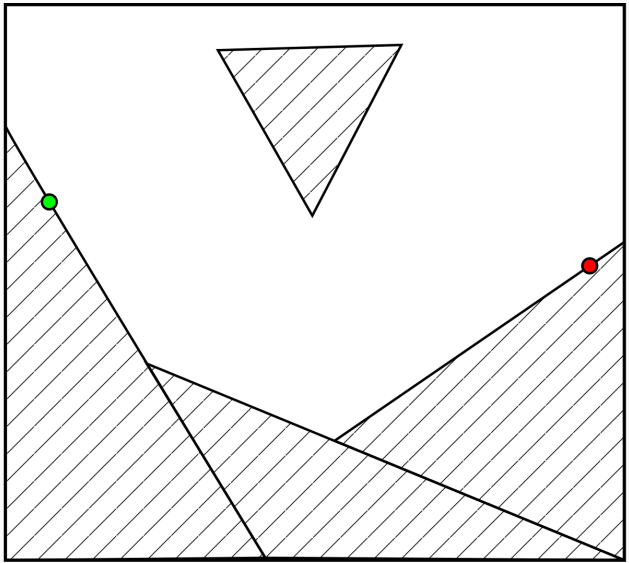

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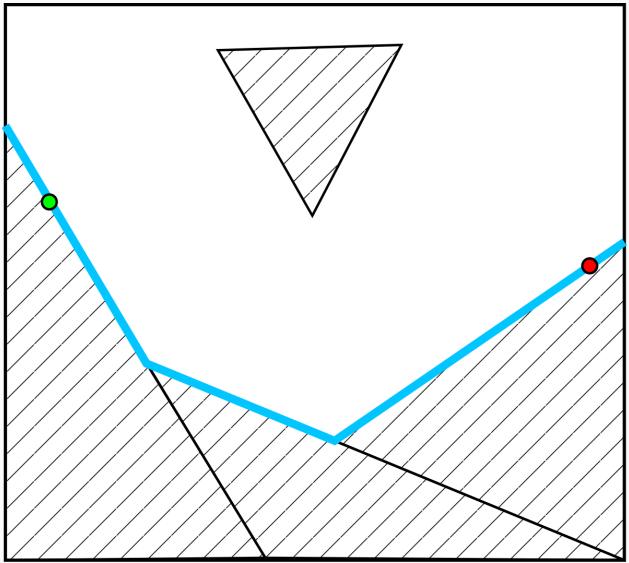
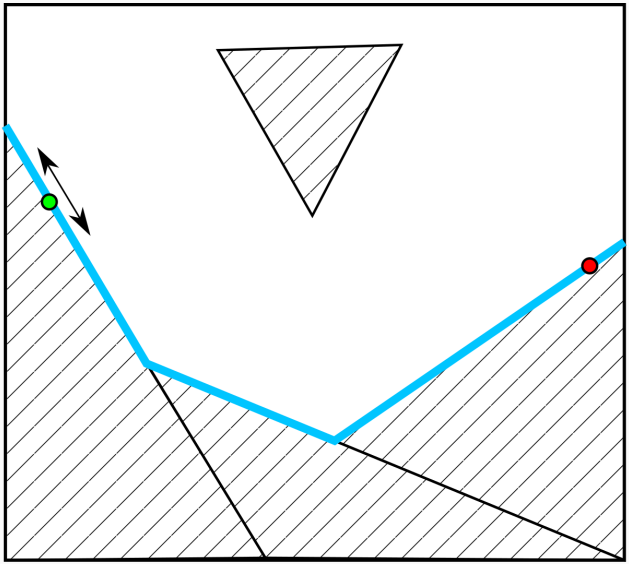


Planning with Manifold Constraints



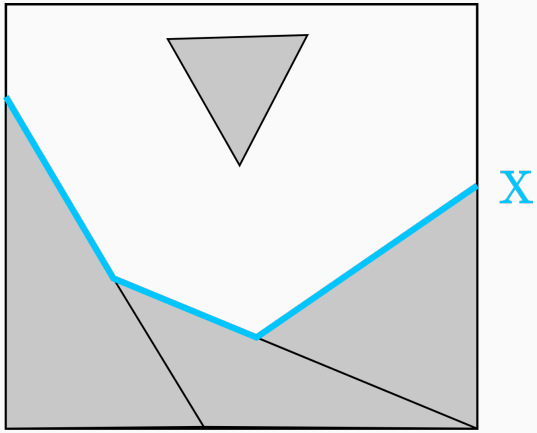
Motion Planning with Manifold Constraints

- Define constraint function $F(q) : Q \to \mathbb{R}^k$ such that $F(q) = 0$ when k constraints are fulfilled.
- This implies that there are $m = n - k$ effective degrees of freedom.
- Constraint function thus defines an m-dimensional constrained configuration space

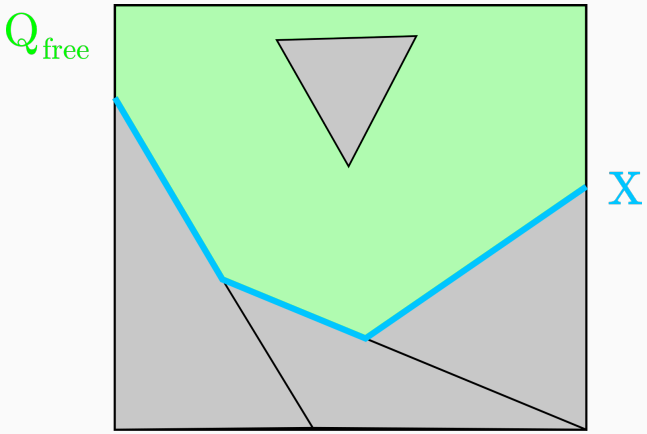
$$X = \{q \in Q \mid F(q) = 0\}$$

- State space Q is called the *ambient space* of X.

Planning with Manifold Constraints



Planning with Manifold Constraints



Planning with Manifold Constraints

Zero-measure constraints

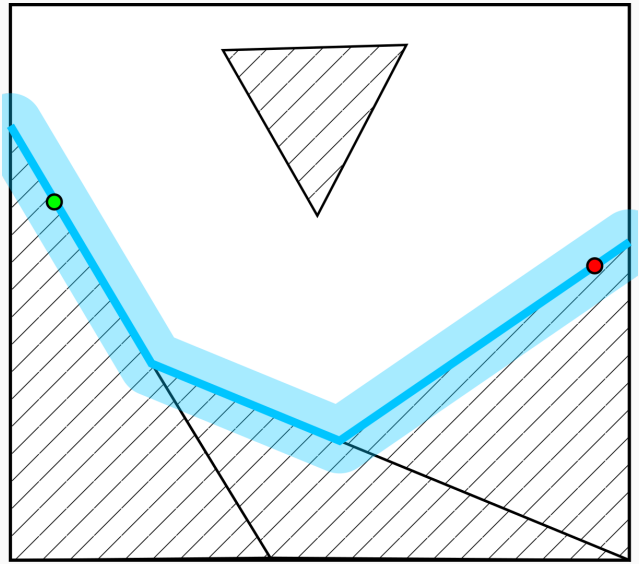
- Sampling not directly possible (zero chance to hit constraints)
- Interpolation not directly possible (will almost always move into ambient space)
- Planning with Manifold Constraints needs to address those two issues.

Planning with Manifold Constraints

Constraint planning approaches

- Relaxation-based
- Projection-based
- Continuation-based

Relaxation-based constraint planning



Relaxation-based constraint planning

- Given some $\epsilon > 0$, define relaxed configuration space

$$X_\epsilon = \{q \in Q \mid \|F(q)\| \leq \epsilon\}$$

- Rejection sampling: Uniform sampling and reject everything outside X_ϵ.

Relaxation-based constraint planning

- Given some $\epsilon > 0$, define relaxed configuration space

$$X_\epsilon = \{q \in Q \mid \|F(q)\| \leq \epsilon\}$$

- Rejection sampling: Uniform sampling and reject everything outside X_ϵ.

Interpolation?

Relaxation-based constraint planning

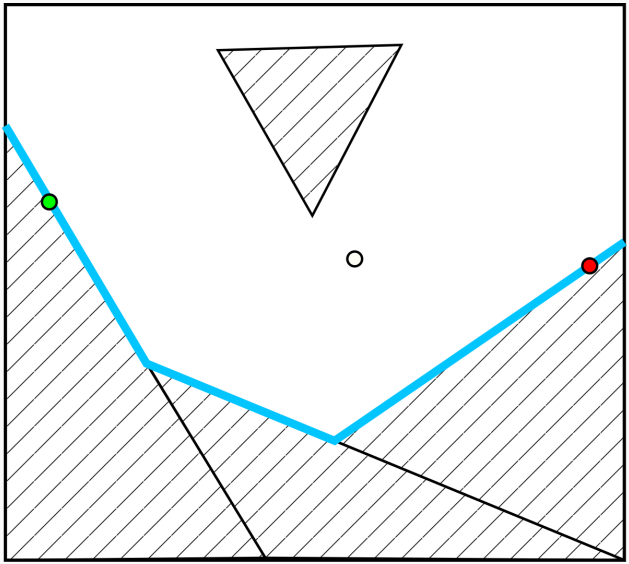
- Given some $\epsilon > 0$, define relaxed configuration space

$$X_\epsilon = \{q \in Q \mid \|F(q)\| \leq \epsilon\}$$

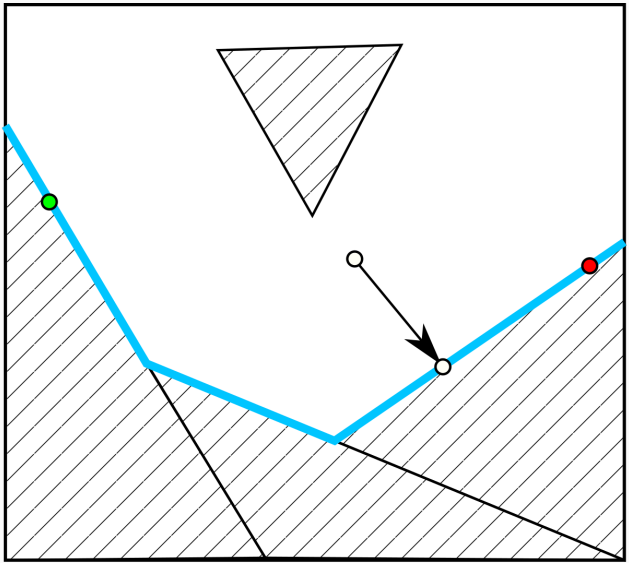
- Rejection sampling: Uniform sampling and reject everything outside X_ϵ.

Probabilistic Completeness?

Projection-based constraint planning



Projection-based constraint planning

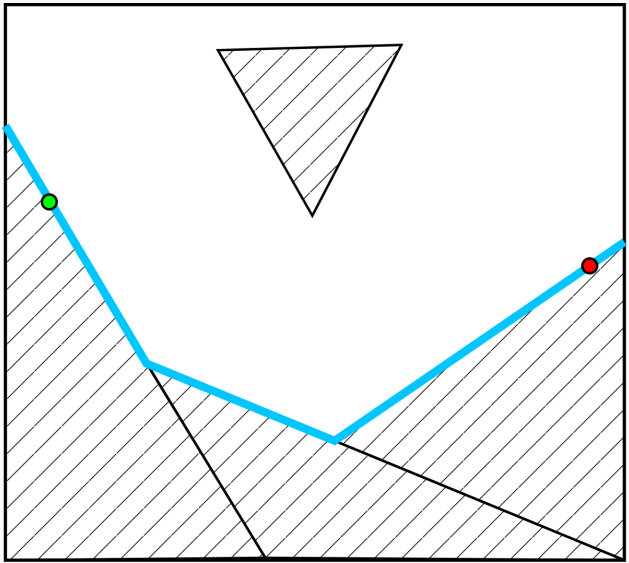


Projection-based constraint planning

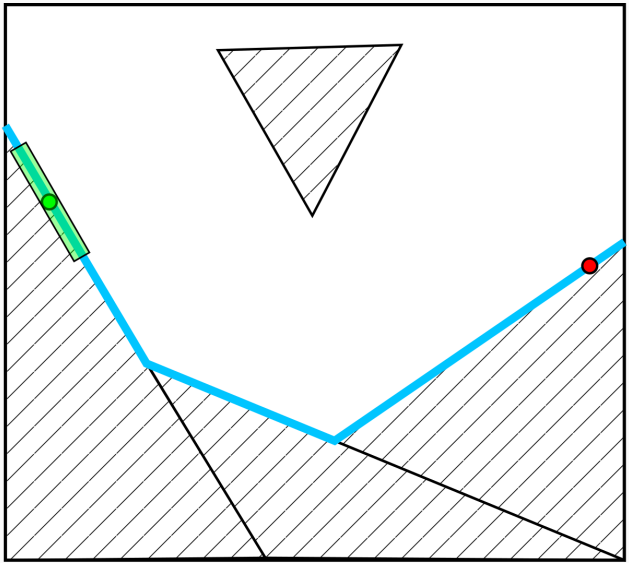
- Given constraint configuration space X and ambient space Q, define projection $\pi : Q \to X$.
- Projection sampling: Uniform sampling and projecting onto X

Interpolation?
Probabilistic Completeness? [**berenson2009manipulation**]

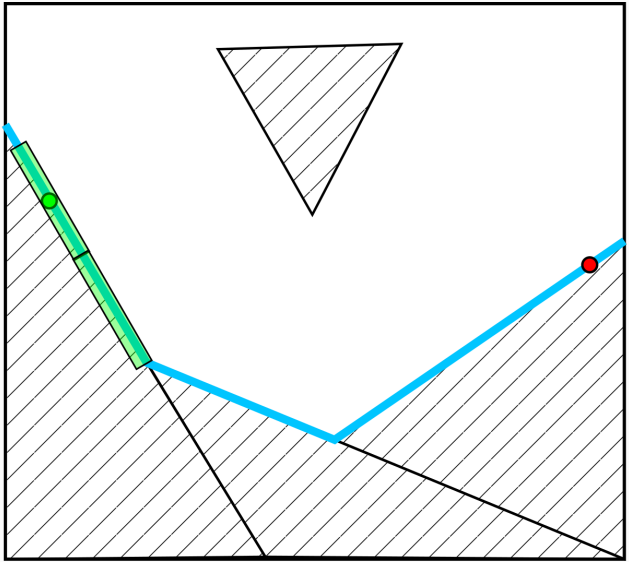
Continuation-based constraint planning



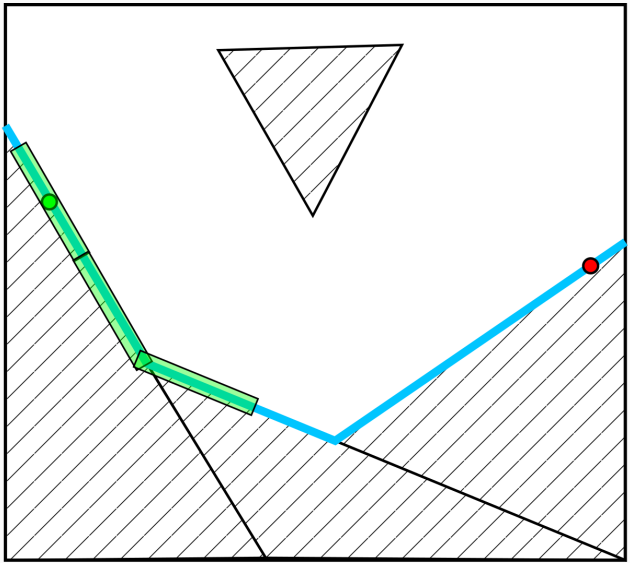
Continuation-based constraint planning



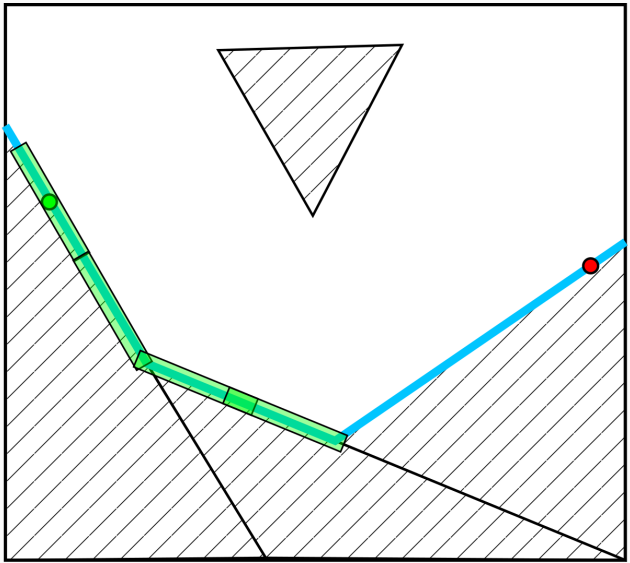
Continuation-based constraint planning



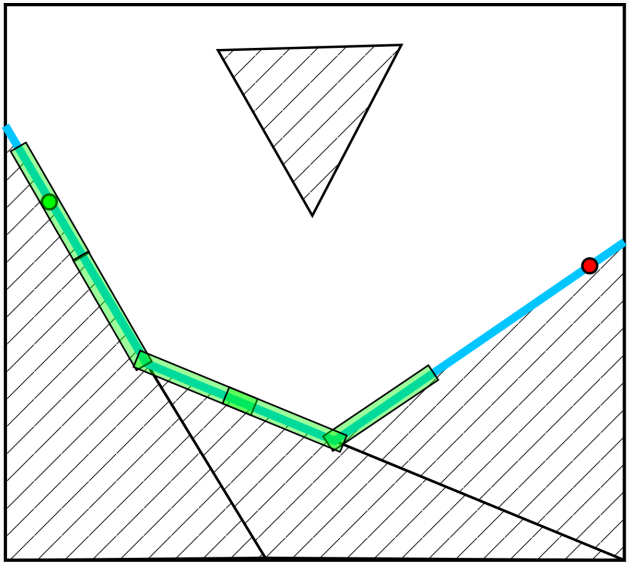
Continuation-based constraint planning



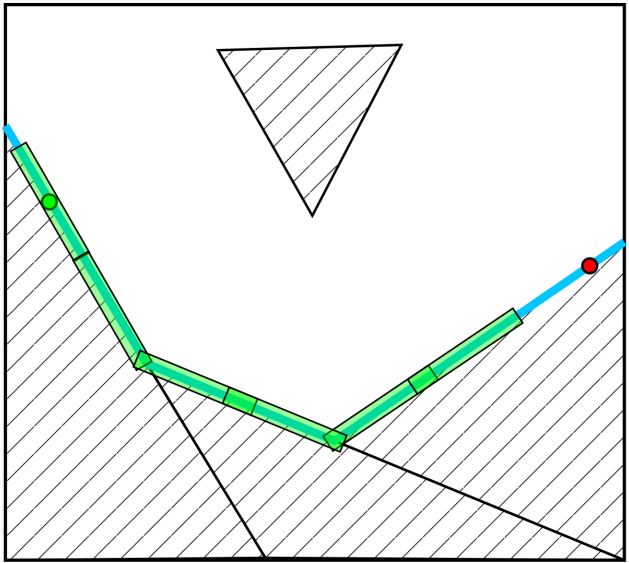
Continuation-based constraint planning



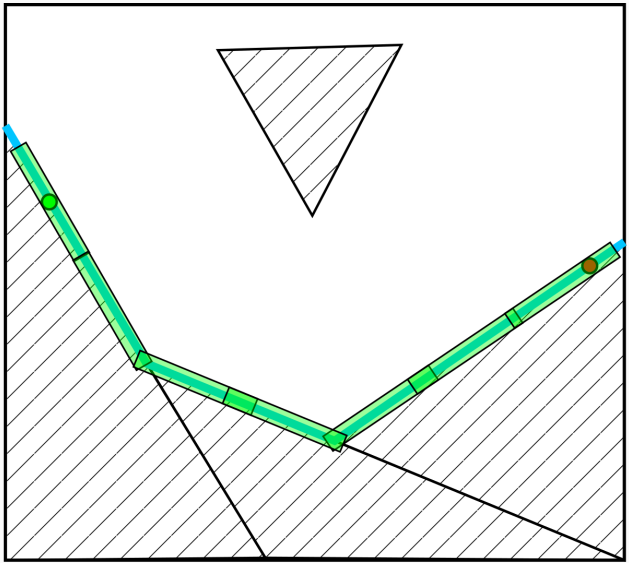
Continuation-based constraint planning



Continuation-based constraint planning



Continuation-based constraint planning

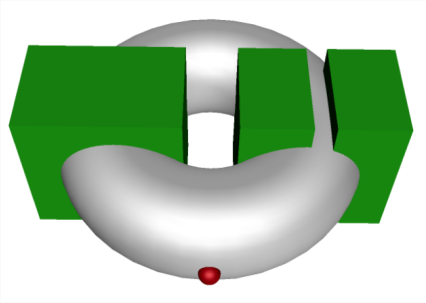


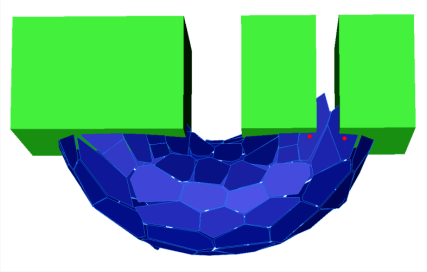
Continuation-based constraint planning

- Compute piecewise-linear approximations (a *chart*) to compute a cover (an *atlas*) on the constraint configuration space Q
- Atlas sampling

Interpolation?
Probabilistic Completeness? [**jaillet2012path**]

Continuation-based constraint planning





BSc/MSc Theses Opportunities

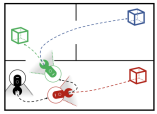
BSc/MSc Theses Opportunities

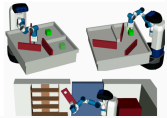
If you found this course interesting, we invite you to write a thesis with us.

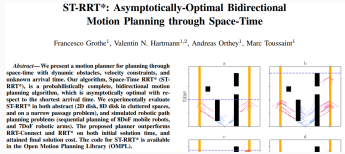
BSc/MSc Theses Opportunities

- Most important requirements: C++ or Python / Linear Algebra
- Wide variety of Topics on Robotics/Motion Planning/Machine Learning
  - Construction Assembly
  - Quadrocopter Flight and Transport
  - Solving Physical Puzzles with Robots
  - Theory of Motion Planning
- Weekly supervision
- Involved in cutting edge research
- Possibility of writing a paper

BSc/MSc Theses Opportunities (Examples papers)







- Janis Eric Freund, et al., Asymptotically Optimal Belief Space Planning in Discrete Partially-Observable Domains, IROS, 2024
- Bora Bayraktar, et al., Solving Rearrangement Puzzles using Path Defragmentation in Factored State Spaces, RAL, 2023
- Francesco Grothe, et al., ST-RRT*: Asymptotically-Optimal Bidirectional Motion Planning through Space-Time, ICRA, 2022
- Marie-Therese Khoury, et al., Efficient Sampling of Transition Constraints for Motion Planning under Sliding Contacts, CASE, 2021

Conclusion

Today

- Multi-robot planning in composite state space
- Projections as requirement for efficient planning
- Homogeneous planning: Pebbles on a graph and conflict-based search
- Non-homogeneous planning: Prioritized vs. Decomposed
- Manifold constraints and zero-measure sets

Exam next week

- Please post questions on the ISIS forum

References i

[1] John Canny. *The complexity of robot motion planning*. MIT press, 1988.

[2] Wenying Wu, Subhrajit Bhattacharya, and Amanda Prorok. "Multi-Robot Path Deconfliction through Prioritization by Path Prospects". In: *arXiv preprint arXiv:1908.02361* (2019).

[3] Daniel Kornhauser, Gary L. Miller, and Paul Spirakis. "Coordinating Pebble Motion on Graphs, The Diameter of Permutation Groups, and Applications". In: *Symposium on the Foundations of Computer Science*. IEEE. Oct. 1984, pp. 241–250.

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[8] Andreas Orthey, Sohaib Akbar, and Marc Toussaint. "Multilevel Motion Planning: A Fiber Bundle Formulation". In: *International Journal of Robotics Research* (2023).

[9] Thierry Siméon, Stéphane Leroy, and Jean P Laumond. "Path coordination for multiple mobile robots: A resolution-complete algorithm". In: *IEEE Transactions on Robotics and Automation* 18.1 (2002), pp. 42–49.

[10] Glenn Wagner and Howie Choset. "Subdimensional expansion for multirobot path planning". In: *Artificial Intelligence* 219 (2015), pp. 1–24.

[11] Kiril Solovey, Oren Salzman, and Dan Halperin. “Finding a needle in an exponential haystack: Discrete RRT for exploration of implicit roadmaps in multi-robot motion planning”. In: *International Journal of Robotics Research* 35.5 (2016), pp. 501–513.