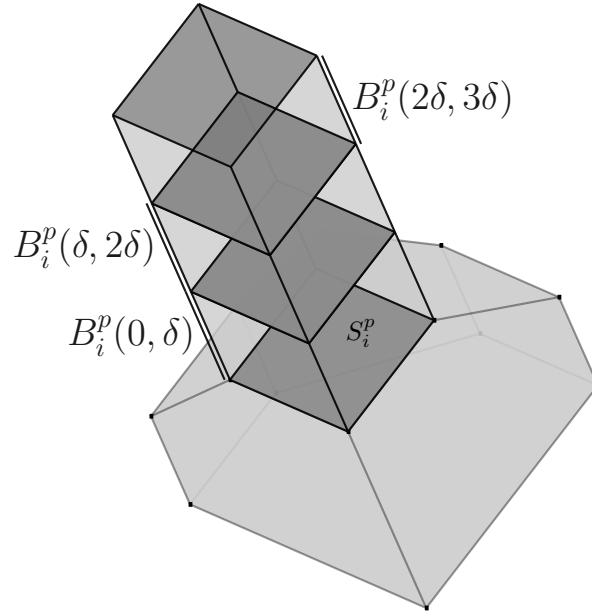


Geometry of Quaders on Top of Surface Elements of Polytopes

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Let us give an explicit expression for a quader which is hovering on top of a surface element of a polytope. First we start with the definition of a polytope [?]:

Definition 1. Let an object O_i be a convex bounded polytope

$$O_i = \{x \in \mathbb{R}^3 | a_j^{(i)T} x \leq b_j^{(i)}, \|a_j^{(i)}\|_2 = 1, j \in [1, M_i]\} \quad (1)$$

with M_i the number of halfspaces.

Let us take one surface element S_i^p of O_i . This is defined as:

Definition 2 (Surface Element). Given an object O_i , we call

$$S_i^p = \{x \in \mathbb{R}^3 | a_p^{(i)T} x = b_p^{(i)}, a_j^{(i)T} x \leq b_j^{(i)}, \\ j = 1, \dots, p-1, p+1, \dots, M_i\} \quad (2)$$

the p -th surface element of object O_i , and $a_p^{(i)}$ is the surface normal with distance $b_p^{(i)}$ to the origin.

A quader on top of S_i^p can now be defined as

Definition 3. The quader $B_i^p(\Delta_L, \Delta_U)$ of height $\delta = \Delta_U - \Delta_L$ located with distance Δ_L above S_i^p is defined as the set of points in

$$\begin{aligned} B_i^p(\Delta_L, \Delta_U) = \{x \in \mathbb{R}^3 | & -a_p^{(i)T} x \leq -b_p^{(i)} - \Delta_L, \\ & a_p^{(i)T} x \leq b_p^{(i)} + \Delta_U, \\ & \hat{a}_j^{(i)T} x \leq \hat{b}_j^{(i)}, \\ & j=1, \dots, p-1, p+1, \dots, M_i \} \end{aligned} \quad (3)$$

with $\hat{a}_j^{(i)}, \hat{b}_j^{(i)}$ belonging to the projected hyperplane j , with

$$\begin{aligned} \hat{a}_j^{(i)} &= a_j^{(i)} - (a_j^{(i)T} a_p^{(i)}) a_p^{(i)} \\ \hat{b}_j^{(i)} &= \hat{a}_j^{(i)T} x_{j,0}^{(i)} \end{aligned} \quad (4)$$

whereby $x_{j,0}^{(i)}$ is one point on the intersection between hyperplane H_j and surface element S_i^p

$$\begin{aligned} x_{j,0}^{(i)} \in \{x \in \mathbb{R}^3 | & a_j^{(i)T} x = b_j^{(i)}, \\ & a_k^{(i)T} y \leq b_k^{(i)}, \\ & k=1, \dots, j-1, j+1, \dots, p-1, p+1, \dots, M_i \}, \\ & a_p^{(i)T} y = b_p^{(i)}, \\ & \|x - y\|^2 = 0 \} \end{aligned} \quad (5)$$

Note that x_0 does only exist, when there is a common border between S_i^p and H_j . If there is no border, then $\hat{a}_j^{(i)}$ and $\hat{b}_j^{(i)}$ do not exist, i.e. they are not halfspace intersections of the box.